To: AP Calculus BC students

From: Mr. Kreuer

Re: Summer work

Directions: All students taking AP Calculus BC next school year should complete all problems contained in the packet. This material will not be covered in class. You are expected to know all the material enclosed before starting Calculus BC. During the summer you can email me to ask any questions about the material contained in the packet. I will check my email semi-regularly and will respond when I can. Don't procrastinate and wait until the last week before school starts to begin completing the packet. My email address is dkreuer@bwschools.net. Students should expect to spend approximately 10 hours on the assignments. The packet should be turned in to your instructor on the first day of school, and a grade will be given based on the completeness and accuracy of solutions. **You must show your work for problems on a separate sheet of paper.**

You should be prepared to take a test on the material covered in this packet when you return to school.

Have a great summer!

Calculus BC

Summer Review Problems

Section 1: Operations with exponents and logarithms.

1) Solve the following. Round your answer to the nearest thousandth.
   a. $\log_3 177$
   b. $\log_{1.8} 3.94$
2) Expand the following using properties of logs.
   a. $\log \frac{x^3 y^4}{z}$
3) Condense the expression to the log of a single quantity.
   a. $\log x - 2\log y + 3\log z$
4) Use the properties of exponents to simplify the following
   a. $\frac{x^5 y^{-2} z^4}{x^2 y^5 z^{.5}}$
   b. $x^4\, x^3$
   c. $(x^2)^4$

Section 2: Trig Functions and their graphs.

1) Graph one period of the six trigonometric functions and identify the following for each.
   a. Domain b. Range c. Period

2) Graph the six inverse trigonometric functions and identify the following for each.

   a. Domain b. Range

Section 3: Trig identities

1) The following identities will be used quite often this year. Please commit them to memory.

a. Pythagorean identities

i. $\sin^2 x + \cos^2 x = 1$ iii. $1 + \tan^2 x = \sec^2 x$

ii. $1 + \cot^2 x = \csc^2 x$

b. Reciprocal identities

i. $\sin x = 1/\csc x$ ii. $\cos x = 1/\sec x$

iii. $\tan x = 1/\cot x$ iv. $\csc x = 1/\sin x$

v. $\sec x = 1/\cos x$ vi. $\cot x = 1/\tan x$

c. Quotient identities

i. $\tan x = \sin x / \cos x$ ii. $\cot x = \cos x / \sin x$

d. Double Angle Formulas

i. $\sin 2x = 2 \sin x \cos x$

ii. $\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$

e. Power Reducing Formulas

i. $sin^2 x = \frac{1-\cos 2x}{2}$ $ii.\ cos^2 x = \frac{1+\cos 2x}{2}$

Section 4: Unit Circle

Fill in the table. Answers should be exact. No decimals.

Degree	Radians	SINE	COS	TAN	CSC	SEC	COT
0							
30							
45							
60							
90							
120							
135							

150							
180							
210							
225							
240							
270							
300							
315							
330							
360							

Section 9: Limits

1) Find the following limits using analytical methods.

a. $\lim_{x\to 2} \frac{x^3-8}{x-2}$

b. $\lim_{x\to 4} \frac{\sqrt{x+5}-3}{x-4}$

c. $\lim_{x\to 4^-} 3[\![x]\!] - 5$

d. $\lim_{x\to\infty} \frac{\sin x}{x}$

e. $\lim_{x\to 9} \frac{x^2-81}{\sqrt{x}-3}$

f. $\lim_{x\to 3^+} \frac{x^2}{x^2-9}$

g. $\lim_{x\to\infty} \frac{x^2-4}{3x^2+8x-1}$

h. $\lim_{x\to 0} 5x^2 \sin\left(\frac{1}{x}\right)$

i. $\lim_{x\to 3^+} \frac{2x+6}{|x+3|}$

j. $\lim_{x\to -\infty} \frac{x}{\sqrt{x^2+6}}$

Section 10: Derivatives

1) Please familiarize yourself with all basic rules of differentiation. You should be able to recall these things from memory. Complete the following.

a. $\frac{d}{dx}(cu)$

b. $\frac{d}{dx}(u \pm v)$

c. $\frac{d}{dx}(uv)$

d. $\frac{d}{dx}\left(\frac{u}{v}\right)$

e. $\frac{d}{dx}(c)$

f. $\frac{d}{dx}(u^n)$

g. $\frac{d}{dx}(x)$

h. $\frac{d}{dx}(\ln u)$

i. $\frac{d}{dx}(e^u)$

j. $\frac{d}{dx}(\log_a u)$

k. $\frac{d}{dx}(a^u)$

l. $\frac{d}{dx}(\sin u)$

m. $\frac{d}{dx}(\cos u)$

n. $\frac{d}{dx}(\tan u)$

o. $\frac{d}{dx}(\cot u)$

p. $\frac{d}{dx}(\sec u)$

q. $\frac{d}{dx}(\csc u)$

2) Find $\frac{dy}{dx}$ for the following.

a. $y = x^2 + 4x - 3$

b. $y = \sin\sqrt{x}$

c. $y = x^2 \cos x$

d. $y = 4(x^3 - 4)^3$

e. $y = \left(\frac{3x^2-1}{2x+5}\right)^3$

f. $x^3 + xy - 4y^{-3} = 2$

g. $y = \ln(e^{x^2})$

h. $2\sin x \cos y = 5$

i. $y = \sin^{-1}(x-1)$

j. $y = \tan(\sin^{-1} x)$

k. $y = (1+x^2)^{\cos x}$

l. $y = 4^{6x}$

3) The displacement of a particle moving in a straight line to north is given by the equation $s(t) = 3t^4 - 8t^3 - 6t^2 + 24t$, where t is in seconds and displacement is in meters.

a) Find the velocity and acceleration after two seconds.

b) When is the particle at rest?

c) When is the particle moving in the south direction?

4) Given $x^3 + y^3 = 6xy$

a) Find $\frac{dy}{dx}$

b) Find the slope of the tangent line at the point (3,3)

c) Find the equation of the tangent line at the point (3,3)

d) Find $\frac{d^2y}{dx^2}$ at the point (3,3)

5) A cylindrical can with a top and a bottom is to have a volume of 128 cubic centimeters. Find the dimensions that will minimize the amount of material required to construct the can.

6) A lighthouse is 100 meters from a straight shoreline. The light turns at a rate of 10 revolution per minute (i.e. 20π radians/minute), and shines a moving spot of light along the shore. How fast is the spot of light moving when it's 100 meters from the point on the shore which is nearest to the light house? Be sure to include the correct units in your answer.