Honors Calculus Summer Assignment

Directions: All students taking Honors Calculus school year should complete all problems contained in the packet. This material will be covered in the first few weeks of class. You are expected to review all the material enclosed before starting school. During the summer you can email me to ask any questions about the material contained in the packet. I will check my email semi-regularly and will respond when I can. Don't procrastinate and wait until the last week before school starts to begin completing the packet. My email address is mhausman@bwschools.net.

You should be prepared to take a test on the material covered in this packet when you return to school.

Have a great summer!

Honors Calculus
Summer Review Problems

Complex Fractions

When simplifying complex fractions, multiply by a fraction equal to 1 which has a numerator and denominator composed of the common denominator of all the denominators in the complex fraction.

Example:

$$\frac{-7-\frac{6}{x+1}}{\frac{5}{x+1}} = \frac{-7-\frac{6}{x+1}}{\frac{5}{x+1}} \bullet \frac{x+1}{x+1} = \frac{-7(x+1)-6}{5} = \frac{-7x-13}{5}$$

$$\frac{\frac{-2}{x}+\frac{3x}{x-4}}{5-\frac{1}{x-4}} = \frac{\frac{-2}{x}+\frac{3x}{x-4}}{5-\frac{1}{x-4}} \bullet \frac{x(x-4)}{x(x-4)} = \frac{-2(x-4)+3x(x)}{5(x)(x-4)-1(x)} = \frac{-2x+8+3x^2}{5x^2-20x-x} = \frac{3x^2-2x+8}{5x^2-21x}$$

Simplify each of the following.

1. $\dfrac{\frac{25}{a}-a}{5+a}$

2. $\dfrac{2-\frac{4}{x+2}}{5+\frac{10}{x+2}}$

3. $\dfrac{4-\frac{12}{2x-3}}{5+\frac{15}{2x-3}}$

4. $\dfrac{\frac{x}{x+1}-\frac{1}{x}}{\frac{x}{x+1}+\frac{1}{x}}$

5. $\dfrac{1-\frac{2x}{3x-4}}{x+\frac{32}{3x-4}}$

Functions

To evaluate a function for a given value, simply plug the value into the function for x.

Recall: $(f \circ g)(x) = f(g(x))$ *OR* $f[g(x)]$ read "***f*** of ***g*** of ***x***" means: plug the inside function (in this case g(x)) in for x in the outside function (in this case, f(x)).

Example: Given $f(x) = 2x^2 + 1$ *and* $g(x) = x - 4$ find *f(g(x))*.

$$\begin{aligned} f(g(x)) &= f(x-4) \\ &= 2(x-4)^2 + 1 \\ &= 2(x^2 - 8x + 16) + 1 \\ &= 2x^2 - 16x + 32 + 1 \\ f(g(x)) &= 2x^2 - 16x + 33 \end{aligned}$$

Let $f(x) = 2x + 1$ *and* $g(x) = 2x^2 - 1$. **Find each.**

6. $f(2) =$ ____________ 7. $g(-3) =$ ____________ 8. $f(t+1) =$ ____________

9. $f[g(-2)] =$ ____________ 10. $g[f(m+2)] =$ ____________ 11. $\dfrac{f(x+h) - f(x)}{h} =$ ______

Let $f(x) = \sin x$ **Find each exactly.**

12. $f\left(\dfrac{\pi}{2}\right) =$ ____________ 13. $f\left(\dfrac{2\pi}{3}\right) =$ ____________

Let $f(x) = x^2$, $g(x) = 2x + 5$, *and* $h(x) = x^2 - 1$. **Find each.**

14. $h[f(-2)] =$ ________ 15. $f[g(x-1)] =$ ________ 16. $g\left[h(x^3)\right] =$ ________

Find $\dfrac{f(x+h) - f(x)}{h}$ **for the given function *f*.**

17. $f(x) = 9x + 3$ 18. $f(x) = 5 - 2x$

Intercepts and Points of Intersection

To find the x-intercepts, let y = 0 in your equation and solve.
To find the y-intercepts, let x = 0 in your equation and solve.

Example: $y = x^2 - 2x - 3$

$x-\text{int. } (Let\ y = 0)$

$0 = x^2 - 2x - 3$

$0 = (x-3)(x+1)$

$x = -1 \text{ or } x = 3$

$x-\text{intercepts } (-1,0) \text{ and } (3,0)$

$y-\text{int. } (Let\ x = 0)$

$y = 0^2 - 2(0) - 3$

$y = -3$

$y-\text{intercept } (0,-3)$

Find the x and y intercepts for each.

19. $y = 2x - 5$

20. $y = x^2 + x - 2$

21. $y = x\sqrt{16 - x^2}$

22. $y^2 = x^3 - 4x$

Use substitution or elimination method to solve the system of equations.
Example:

$$x^2 + y^2 - 16x + 39 = 0$$
$$x^2 - y^2 - 9 = 0$$

<u>Elimination Method</u>

$2x^2 - 16x + 30 = 0$

$x^2 - 8x + 15 = 0$

$(x-3)(x-5) = 0$

$x = 3 \text{ and } x = 5$

Plug $x = 3$ *and* $x = 5$ into one original

$3^2 - y^2 - 9 = 0 \qquad 5^2 - y^2 - 9 = 0$

$-y^2 = 0 \qquad 16 = y^2$

$y = 0 \qquad y = \pm 4$

Points of Intersection $(5,4)$, $(5,-4)$ *and* $(3,0)$

<u>Substitution Method</u>

Solve one equation for one variable.

$y^2 = -x^2 + 16x - 39$ (1st equation solved for y)

$x^2 - (-x^2 + 16x - 39) - 9 = 0$ Plug what y^2 is equal to into second equation.

$2x^2 - 16x + 30 = 0$ (*The rest is the same as previous example*)

$x^2 - 8x + 15 = 0$

$(x-3)(x-5) = 0$

$x = 3 \text{ or } x - 5$

Find the point(s) of intersection of the graphs for the given equations.

23. $x + y = 8$, $4x - y = 7$

24. $x^2 + y = 6$, $x + y = 4$

25. $2x - 3y = 5$, $5x - 4y = 6$

<u>Interval Notation</u>

26. Complete the table with the appropriate notation or graph.

Solution	Interval Notation	Graph
$-2 < x \le 4$		
	$[-1, 7)$	
		

Equation of a line

Slope intercept form: $y = mx + b$

Vertical line: x = c (slope is undefined)

Point-slope form: $y - y_1 = m(x - x_1)$

Horizontal line: y = c (slope is 0)

38. Use slope-intercept form to find the equation of the line having a slope of 3 and a y-intercept of 5.

39. Determine the equation of a line passing through the point (5, -3) with an undefined slope.

40. Determine the equation of a line passing through the point (-4, 2) with a slope of 0.

41. Use point-slope form to find the equation of the line passing through the point (0, 5) with a slope of 2/3.

42. Find the equation of a line passing through the point (2, 8) and parallel to the line $y = \frac{5}{6}x - 1$.

43. Find the equation of a line perpendicular to the y- axis passing through the point (4, 7).

44. Find the equation of a line passing through the points (-3, 6) and (1, 2).

45. Find the equation of a line with an x-intercept (2, 0) and a y-intercept (0, 3).

Reference Triangles (If you don't remember how to do this, google "reference triangles")

49. Sketch the angle in standard position. Draw the reference triangle and label the sides, if possible.

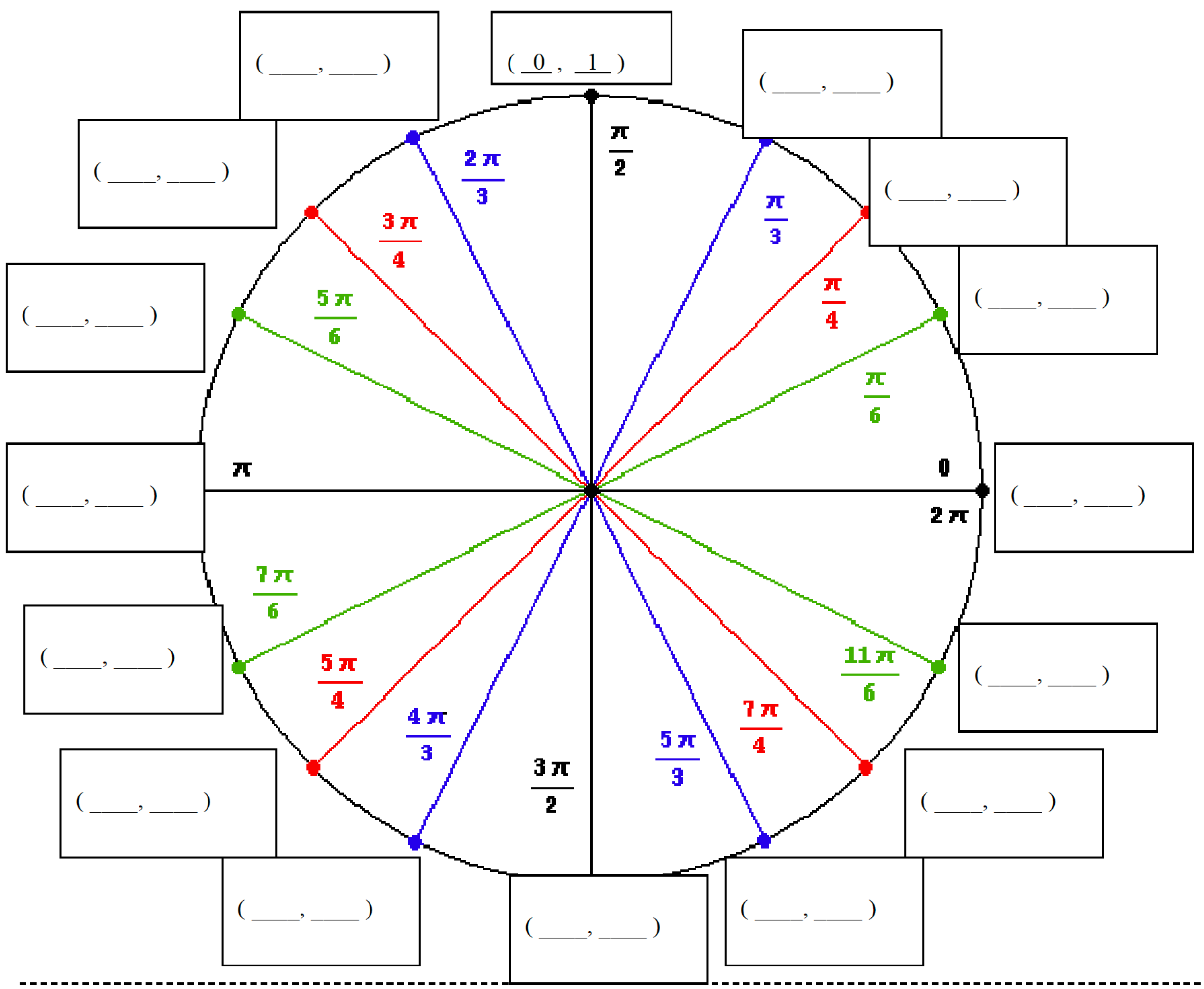
a. $\frac{2}{3}\pi$

b. 225°

c. $-\frac{\pi}{4}$

d. 30°

Unit Circle Worksheet



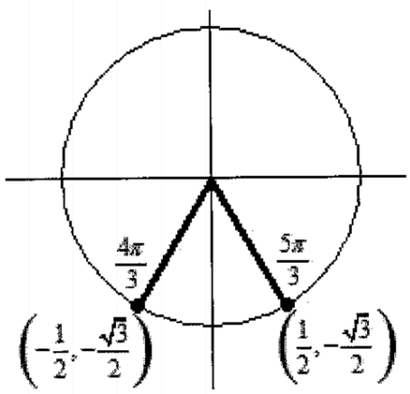
Trigonometric Equations
We'll have to solve some trigonometric equations incidentally throughout calculus. Here's a quick review on how the unit circle can help solve these equations.

Ex) Solve the equation $\sqrt{3}\cdot\csc x+2=0$ on the interval $[0,2\pi)$.
SOLUTION:

First: Solve for the trig function involved →

$$\sqrt{3}\cdot\csc x+2=0$$
$$\sqrt{3}\cdot\csc x=-2$$
$$\frac{\cancel{\sqrt{3}}\cdot\csc x}{\cancel{\sqrt{3}}}=-\frac{2}{\sqrt{3}}$$
$$\csc x=-\frac{2}{\sqrt{3}}$$

If $\csc x=-\frac{2}{\sqrt{3}}$ then this also means $\sin x=-\frac{\sqrt{3}}{2}$ ← this will help you identify the angles on the unit circle.



Second: Identify the angles which satisfy the equation:

Sine is defined by as the y-coordinates on the unit circle. You're looking for the unit circle angles where the y-coordinate is $-\sqrt{3}/2$.

This happens in two places $x=\frac{4\pi}{3}$ and $x=\frac{5\pi}{3}$.

The solutions are $x=\frac{4\pi}{3}$ and $x=\frac{5\pi}{3}$.

50. $3\sin\theta+2\sin^2\theta=-1$

51. $\cos^2\theta=2+2\sin\theta$

52. $4\sin^2\theta+4=5$

53. $\sec^2\theta-6=-4$

Know these…

Formula Sheet

Reciprocal Identities: $\csc x = \dfrac{1}{\sin x}$ $\sec x = \dfrac{1}{\cos x}$ $\cot x = \dfrac{1}{\tan x}$

Quotient Identities: $\tan x = \dfrac{\sin x}{\cos x}$ $\cot x = \dfrac{\cos x}{\sin x}$

Pythagorean Identities: $\sin^2 x + \cos^2 x = 1$ $\tan^2 x + 1 = \sec^2 x$ $1 + \cot^2 x = \csc^2 x$

Double Angle Identities: $\sin 2x = 2\sin x \cos x$

$\tan 2x = \dfrac{2\tan x}{1 - \tan^2 x}$

$$\begin{aligned} \cos 2x &= \cos^2 x - \sin^2 x \\ &= 1 - 2\sin^2 x \\ &= 2\cos^2 x - 1 \end{aligned}$$

Logarithms: $y = \log_a x$ is equivalent to $x = a^y$

Product property: $\log_b mn = \log_b m + \log_b n$

Quotient property: $\log_b \dfrac{m}{n} = \log_b m - \log_b n$

Power property: $\log_b m^p = p \log_b m$

Property of equality: If $\log_b m = \log_b n$, then m = n

Change of base formula: $\log_a n = \dfrac{\log_b n}{\log_b a}$

Slope-intercept form: $y = mx + b$

Point-slope form: $y - y_1 = m(x - x_1)$

Standard form: Ax + By + C = 0